

Turbulent acoustic streaming excited by resonant gas oscillation with periodic shock waves in a closed tube

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Abstract: Resonant gas oscillations with periodic shock waves in a closed tube are studied by executing large-scale computations of the compressible 2-D Navier–Stokes equations with a finite-difference method. In a quasi-steady state of oscillation, acoustic streaming (mean mass flow) of large Rs is excited, where Rs is the streaming Reynolds number based on a characteristic streaming velocity, the tube length, and the kinematic viscosity. When $Rs = 560$, relatively strong vortices are localized near the tube wall. The resulting streaming pattern is almost stationary but quite different from that of the Rayleigh streaming. The streaming of $Rs = 6200$ involves unsteady vortices in a region near the center of the tube. Turbulent streaming appears in the result of $Rs = 56000$, where vortices of various scales are irregularly generated throughout the tube.

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Introduction

We shall consider the streaming motions excited by fundamental resonant gas oscillation in a two-dimensional closed tube filled with an ideal gas. The tube, whose length is L and width is W , is closed at one end by a solid plug and the other by a piston (sound source) oscillating harmonically with an amplitude a and angular frequency ω (see Fig. 1). When the source frequency is in a narrow band around a resonant frequency, the resulting gas oscillation may not be a sinusoidal standing wave with fixed loops and nodes but a nonlinear oscillation of large amplitude including periodic shock waves traveling in the tube repeatedly reflected at the sound source and closed end.¹

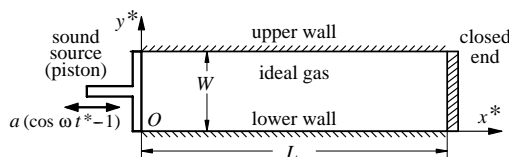


Fig. 1. Schematic of the model.

Such large-amplitude oscillations can induce streaming motions of large streaming Reynolds number $Rs = U_s L_s / \nu$, where U_s is a characteristic magnitude of streaming, L_s is a linear dimension of the system, and ν is the kinematic viscosity. It is known that the jet-like streaming of $Rs \gg 1$ usually becomes a turbulent flow.^{2,3} However, the excitation of turbulent streaming in a resonant tube has remained unsolved. In the present study, we shall numerically demonstrate when and how the streaming motion in the tube becomes turbulent. Because very large-scale (extensive) computations are required, we restrict ourselves to the case in which the angular frequency at the sound source, ω , is equal to the fundamental resonance (angular) frequency $c_0 \pi / L$, where c_0 is the speed of sound in an initially undisturbed gas.

In practical applications of high-intensity resonant oscillations in a closed tube,⁴ the excitation of large Rs streaming may often be inevitable. The knowledge of classical streaming induced by the linear sinusoidal standing wave with fixed loops and nodes is useless because

there should be a steady creeping motion of $Rs < 1$.⁵⁻⁷ An understanding of large Rs and turbulent streaming may be indispensable for the development of such applications.

Problem

We shall consider the fundamental resonance in a closed tube in a case of a wide tube and low frequency. That is, the tube length L satisfies the condition

$$L\omega/c_0 = \pi, \quad (1)$$

and the width W is sufficiently large compared with the typical dimension of the Stokes boundary layer on the wall,

$$W\omega/c_0 = A\pi \gg \epsilon = \sqrt{\nu\omega}/c_0, \quad (2)$$

where $A = W/L$ is an aspect ratio of the tube, and ϵ is a normalized typical linear dimension of the Stokes boundary layer on the wall.

The sound source is a piston located at $x = 0$ for $t < 0$, which begins oscillating harmonically with amplitude a and angular frequency ω at $t = 0$, where $x = x^*\omega/c_0$ and $t = \omega t^*$. The acoustic Mach number at the source, M , is supposed to be sufficiently small compared with unity,

$$M = a\omega/c_0 \ll 1. \quad (3)$$

In the present study, we assume that the order of M is comparable with that of ϵ , i.e.,

$$\alpha = \epsilon/M = O(1), \quad (4)$$

where α is a nondimensional constant. The acoustic Reynolds number at the sound source may then be given as

$$Re = 2\pi/\epsilon\alpha, \quad (5)$$

which is sufficiently large compared with unity. In addition to condition $Re \gg 1$, if the dispersion and attenuation effects⁸ due to the Stokes boundary layer are sufficiently small, a shock with a discontinuous wave front will be formed. The dispersion effect can be estimated by a nondimensional parameter $\epsilon/(A\sqrt{M})$, because, as shown below (see also Ref. 1), a nondimensional wave amplitude at a quasi-steady state (an almost steady state of gas oscillation) is of $O(\sqrt{M})$. Conditions (3) and (4) ensure that the wave amplitude increases to $O(\sqrt{M})$. By the second-order nonlinear effect, the streaming of $O(M)$ is induced, whose Reynolds number Rs is large so that streaming may exhibit a transition from laminar to turbulent.

Under conditions (1)–(4), we shall numerically solve the initial- and boundary-value problem of the two-dimensional Navier–Stokes equations for compressible flow. We assume that the temperature on the solid surface is constant. The gas in the tube is considered to be air (the ratio of specific heats is 1.4 and the Prandtl number is 0.7). Sutherland's formula is adopted for the temperature dependence of shear viscosity, and the bulk viscosity is neglected for simplicity. The flow field is supposed to be symmetric around $y = A\pi/2$, where $y = y^*\omega/c_0$.

Numerical method

We have to use a numerical method capable of resolving discontinuous shock waves. Therefore an upwind finite-difference TVD scheme⁹ is employed, because the capability of the method has already been confirmed in the analysis of the near field of oscillating circular piston.¹⁰ The 2-D Navier–Stokes equations are directly solved without introducing further assumptions. The turbulent streaming motion is not artificially excited but self-generated in the numerical solution for the case of sufficiently large Rs .

In many cases, the two-dimensional assumption is suitable for gas oscillation but, in reality, the excited turbulent motion may have a three-dimensional characteristic. Nevertheless, we can obtain valuable information from 2-D direct simulations. The execution of 3-D simulations would be extraordinarily expensive for the present problem.

The lower half of the tube, $M(\cos t - 1) \leq x \leq \pi$ and $0 \leq y \leq A\pi/2$, is subdivided into a 300×60 nonuniform mesh, where the minimum grid size is less than $\epsilon/4$. Mesh points

are clustered near the solid surface, and hence we can resolve the Stokes boundary layer and a secondary boundary layer of thickness of $O(1/\sqrt{Rs})$.^{2,3} The time step is $2\pi/120000$, and the CFL number is about 0.5. To clarify the transition process to turbulent streaming, very lengthy computations are required. For example, the cpu time for 250 cycles of piston oscillation exceeds 200 hours on the supercomputer at Hokkaido University.

As a preliminary test, we have calculated the case of $M = 0.0004$ and $\epsilon^2 = 0.0001$ ($Re = 25$), where shock waves are not formed, and a steady creep flow of $Rs = 0.5$ is induced. The streaming motion quantitatively agrees with those in Refs. 5–7. The results presented in the following have been validated by verifying the convergence that resulted from comparing the results obtained with different mesh sizes.

The important parameters that characterize the present problem are the source Mach number M , the normalized thickness of the Stokes layer, ϵ , and the aspect ratio A : $A = 0.1$ and ϵ is chosen as 4.5×10^{-4} . The latter corresponds to the source frequency $\omega/2\pi = 250\text{Hz}$ in the air of the standard state. We have computed three cases of $M = 0.000036, 0.0004$, and 0.0036 . The parameters and results are summarized in Table 1. Short animations of main results can be seen at URL: <http://www.hucc.hokudai.ac.jp/~b11422>.¹¹

Table 1. Parameters and main results.

Mach number at Source	SPL* (Source)	SPL** (Max)	$Rs = \frac{\pi}{\alpha\epsilon}$	$\frac{\epsilon}{A\sqrt{M}}$	Shock	Streaming
0.000036	104.9dB	147.5dB	560	0.75	finite thickness	stationary flow pattern
0.0004	125.8dB	161.9dB	6200	0.23	discontinuity	unsteady
0.0036	144.9dB	172.4dB	56000	0.08	discontinuity	turbulent

* SPL for the plane progressive sinusoidal wave radiated by the corresponding sound source.

** SPL based on the rms value of pressure perturbation at closed end in the quasi-steady state.

Resonant gas oscillation with periodic shock waves

First, we shall present the evolution of on-axis velocity amplitude from the initial state of uniform and at rest (Fig. 2). The amplitude initially grows in proportion to Mt . At a large t of $O(1/\sqrt{M})$, an almost steady state (quasi-steady state) is established, where the maximum amplitude of oscillation during one period is almost constant of $O(\sqrt{M})$. The quasi-steady state is supported mainly because of the balance of energy input at the source and energy dissipation at the shock front.

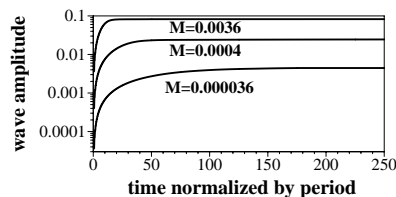


Fig. 2. Evolution of velocity amplitude on the symmetric axis, which is evaluated as the maximum of u minus the minimum of u at $t = n\pi/2$ ($n = 0, 1, 2, \dots$), where $u = u^*/c_0$ is the axial component of nondimensionalized fluid velocity.

Figure 3 shows the wave profiles in the quasi-steady state. A wire-framed yellow disk in the figure is merely a virtual image of the sound source. Note that our computations are neither three-dimensional nor axisymmetric. Since $\epsilon/(A\sqrt{M})$ is not so small for $M = 0.000036$ (see Table 1), the dispersion effect due to boundary layer prevents the shock front from becoming steepened [Figs. 3(a) and 3(b)]. For the cases of $M = 0.0004$ and 0.0036 , the shock front develops into a discontinuity. From Figs. 3(a), 3(c), and 3(e), one can readily see that the profile of axial fluid velocity has a small peak in the boundary layer (Richardson's annular effect).

Roughly speaking, the fluid motion outside the boundary layer can be regarded as the superposition of resonant oscillation and streaming motion. Accordingly, in the case in which

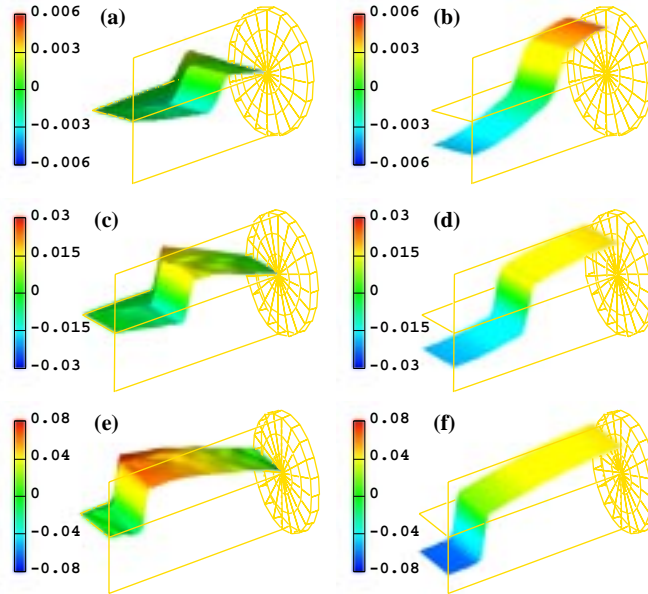


Fig. 3. Snapshots of wave profiles of the normalized axial velocity $u = u^*/c_0$ and sound pressure $p = (p^* - p_0)/\rho_0 c_0^2$. (a) u and (b) p for $M = 0.000036$ at $t = 879.5\pi$, (c) u and (d) p for $M = 0.0004$ at $t = 589.5\pi$, and (e) u and (f) p for $M = 0.0036$ at $t = 461.5\pi$. Each color bar indicates the value of u or p .

the streaming velocity is relatively large and irregular (see Figs. 4 and 5), the axial velocity outside the boundary layer is slightly uneven as shown in Figs. 3(c) and 3(e). Since entropy (and also vorticity) is convected by streaming, the profiles of density and temperature also possess the same unevenness. The pressure profile, on the other hand, is hardly affected by the boundary layer and streaming, and hence is almost independent of the distance from the lower wall.

Excitation of turbulent streaming

The normalized velocity of acoustic streaming (mean mass flow) is defined as

$$\mathbf{u}_S = \begin{pmatrix} u_S \\ v_S \end{pmatrix} = \begin{pmatrix} \overline{\rho u} \\ \overline{\rho v} \end{pmatrix} = \frac{1}{2\pi} \int_{t-2\pi}^t \begin{pmatrix} \rho u \\ \rho v \end{pmatrix} dt, \quad (6)$$

where $\rho = \rho^*/\rho_0$ is a normalized density, $u = u^*/c_0$ and $v = v^*/c_0$ are x and y components of the normalized fluid velocity, and the bar denotes the time average. A typical streaming velocity in each case is of $O(M)$, i.e., the square of the maximum fluid velocity of $O(\sqrt{M})$. The nominal streaming Reynolds number Rs can therefore be estimated as $\pi/\alpha\epsilon$ (see Table 1). The actual maximum speeds of \mathbf{u}_S shown in Figs. 4(a)–4(c) are 1.3cm/s, 59cm/s, and 175cm/s, respectively.

We have confirmed numerically that, as in the linear standing wave problem, u_S and v_S are nearly equal to \bar{u} and \bar{v} because oscillation of ρ is out of phase with that of fluid velocity and hence the so-called velocity transform⁷ is small compared with the magnitude of \mathbf{u}_S . Because the magnitude of $\text{div } \mathbf{u}_S$ is sufficiently small almost everywhere compared with that of \mathbf{u}_S , streaming behaves like a viscous incompressible fluid flow. The perturbation of time-averaged density $\bar{\rho} - 1$ is convected by the streaming motion (the time-averaged pressure is almost independent of it).

The color contours in Fig. 4 indicate the distribution of $\text{curl } \mathbf{u}_S$. In the case of $M = 0.000036$ ($Rs = 560$), the streaming pattern shown in Fig. 4(a) is almost invariant from $t = 440\pi$ to at least $t = 918\pi$. Because Rs is not small, vorticity in the Stokes layer is hardly diffused and, in addition, the streaming velocity is not large enough to propagate the vorticity in

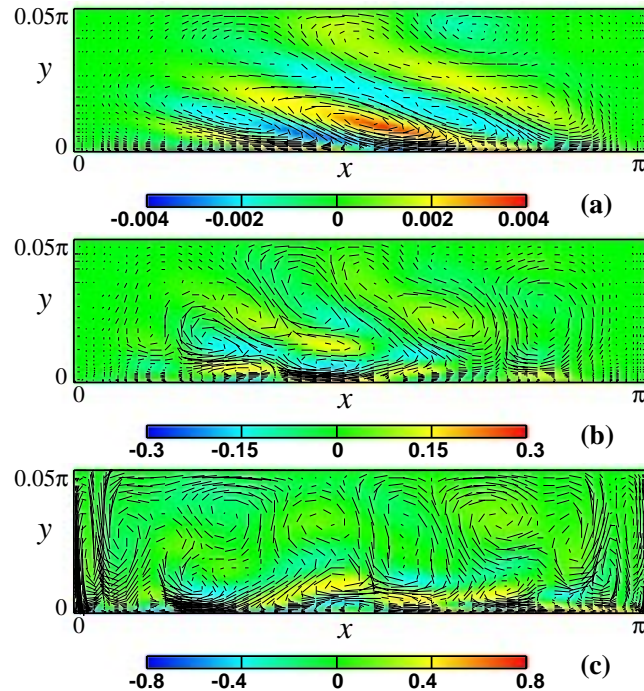


Fig. 4. Streaming patterns. Line segments represent the streaming velocity vector. The color bar denotes the strength of vorticity of streaming. (a): $M = 0.000036$ at $t = 918\pi$ ($Rs = 560$), (b): $M = 0.0004$ at $t = 628\pi$ ($Rs = 6200$), (c): $M = 0.0036$ at $t = 500\pi$ ($Rs = 56000$).

the vicinity of the wall to everywhere in the tube. As a result, some strong vortices are localized near the wall, and the flow pattern in Fig. 4(a) is quite different from that of the classical slow streaming of $Rs < 1$ excited by the linear standing wave.⁵⁻⁷ Figure 4(b) shows the streaming pattern for $M = 0.0004$ ($Rs = 6200$), in which several unsteady vortices are produced. They are confined to a region near the center of the tube, at least up to $t = 628\pi$. Turbulent streaming appears in the case of $M = 0.0036$ ($Rs = 56000$), where irregular and unsteady vortices of various scales are produced throughout the tube [see Fig. 4(c) and Fig. 5(c)]. The turbulent motion causes the time-averaged density and temperature to fluctuate irregularly.

Figure 5 shows the temporal evolution of u_S/M . Compared with Fig. 2, one can see that u_S at $x \cong \pi/2$ grows abruptly when the oscillation attains the quasi-steady state. We here notice that u_S is small at $x \cong \pi/2$ in the classical streaming of $Rs < 1$.⁵⁻⁷ The actual maximum speeds of u_S in Figs. 5(a)–5(c) are 1.2cm/s, 81cm/s and 200cm/s, respectively.

In the case of $Rs = 560$, although the streaming pattern shown in Fig. 4(a) is almost stationary, the local streaming velocity shown in Fig. 5(a) gradually varies for $t > 500\pi$. The axial streaming velocity for $Rs = 6200$ in Fig. 5(b) is unsteady after the oscillation has reached a quasi-steady state. However, we cannot examine whether the fluctuation of streaming is irregular because the numerical result for $Rs = 6200$ is limited to $t \leq 628\pi$; the required computation to answer the question is too large to be executed. In the case of $Rs = 56000$ [Fig. 5(c)], u_S fluctuates irregularly throughout the tube for $t > 60\pi$.

Conclusions

We have demonstrated numerically the excitation of turbulent acoustic streaming by resonant gas oscillation in a closed tube. When $M \ll 1$ and $\epsilon = O(M)$ (and hence $Re \gg 1$), shock waves are formed and the resonant gas oscillation attains a quasi-steady state at a large t of $O(1/\sqrt{M})$, where the normalized wave amplitude is of $O(\sqrt{M})$. The magnitude of the resulting acoustic streaming, which is induced by the second-order nonlinear effect of gas oscillation,

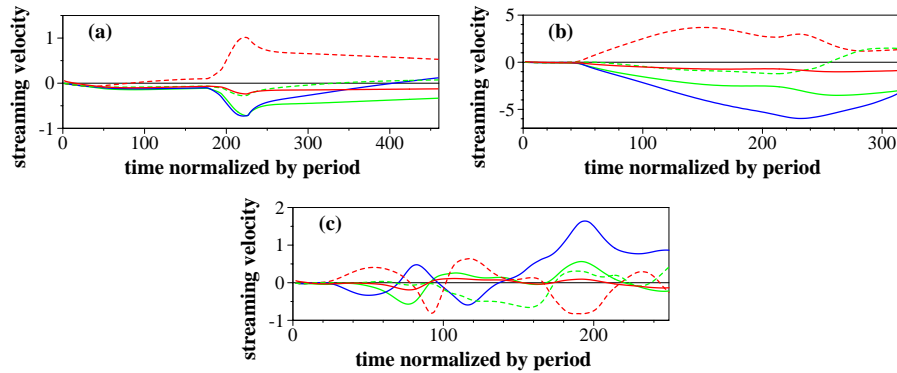


Fig. 5. Time evolution of u_s/M at $x = 1.4084$ and $y = 0.0012$ (red solid curve), $y = 0.0043$ (green solid curve), $y = 0.0130$ (blue solid curve), $y = 0.0348$ (red dashed curve), and $y = 0.0821$ (green dashed curve). (a): $M = 0.000036$ ($Rs = 560$), (b): $M = 0.0004$ ($Rs = 6200$), (c): $M = 0.0036$ ($Rs = 56000$).

is of $O(M)$, namely the streaming velocity is of the same order of magnitude as the piston velocity at the sound source. Accordingly, Rs is as large as Re . The instability of large Rs flow leads to the occurrence of turbulent acoustic streaming.

Finally, we conclude that, in the present computations, the Reynolds number based on the thickness of the Stokes layer is of $O(1)$, and streaming in the Stokes layer is disturbed but, as a whole, remains laminar. If the Reynolds number based on the Stokes layer thickness exceeds its transition Reynolds number, the oscillation in the Stokes layer itself will become turbulent, and the turbulence will occur in periodic bursts followed by a relaminarization in the same cycle of oscillation.¹²

Acknowledgments

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References and links

- ¹ W. Chester, "Resonant oscillations in closed tubes," *J. Fluid Mech.* **18**, 44–65 (1964).
- ² J. T. Stuart, "Double boundary layers in oscillatory viscous flow," *J. Fluid Mech.* **24**, 673–687 (1966).
- ³ M. J. Lighthill, "Acoustic streaming," *J. Sound Vib.* **61**, 391–418 (1978).
- ⁴ See, for example, G. W. Swift, "Thermoacoustic engines and refrigerators," *Phys. Today* **48**, 22–28 (July 1995). See, also, C. C. Lawrenson, B. Lipkens, T. S. Lucas, D. K. Perkins, and T. W. Van Doren, "Measurements of macrosonic standing waves in oscillating closed cavities," *J. Acoust. Soc. Am.* **104**, 623–636 (1998).
- ⁵ Lord Rayleigh, *The Theory of Sound* (Dover, New York, 1945).
- ⁶ W. L. Nyborg, "Acoustic streaming due to attenuated plane waves," *J. Acoust. Soc. Am.* **25**, 68–75 (1953).
- ⁷ P. J. Westervelt, "The theory of steady rotational flow generated by a sound field," *J. Acoust. Soc. Am.* **25**, 60–67 (1953).
- ⁸ M. F. Hamilton, Y. A. Il'inskii, and E. A. Zabolotskaya, "Dispersion," in *Nonlinear Acoustics*, edited by M. F. Hamilton and D. T. Blackstock (Academic Press, San Diego, 1998) pp. 151–175.
- ⁹ S. R. Chakravarthy, "Development of upwind schemes for the Euler equations," *NASA Contractor Report* 4043 (1987).
- ¹⁰ T. Yano and Y. Inoue, "Strongly nonlinear waves and streaming in the near field of a circular piston," *J. Acoust. Soc. Am.* **99**, 3353–3372 (1996).
- ¹¹ T. Yano, "Turbulent acoustic streaming," <http://www.hucc.hokudai.ac.jp/~b11422>.
- ¹² P. Merkli and H. Thomann, "Transition to turbulence in oscillating pipe flow," *J. Fluid Mech.* **68**, 567–575 (1975).