Notes for the slides:

1. Nonneutral plasmas occupy a rather small corner of plasma physics but have been surprisingly successful in making connections to the wider world of physics, and today I would like to discuss some examples of connections to atomic physics, condensed matter physics, astrophysics, and fluid dynamics.

2. A first example shows how laser cooled pure ion plasmas are used to model the microscopic order, or correlation, in high-density matter (e.g., in a white dwarf). This example combines theoretical ideas from condensed matter physics and astrophysics with experimental techniques from plasma physics and atomic physics.

   A second example shows how laser cooled pure ion plasmas are used to model thermonuclear fusion in high-density matter, that is, to measure the Salpeter enhancement factor of the fusion rate.

   A third example shows how pure electron plasmas are used to model the 2D vortex dynamics and turbulence of an ideal fluid (an incompressible and inviscid fluid).

   An example that I won't have time to talk about is the use of cryogenic positron and antiproton plasmas to produce antihydrogen (the ATRAP and ALPHA collaborations at CERN). The goal of this work is to carry out fundamental physics studies with the antihydrogen, but the methods used to obtain the antihydrogen involve a combination of nonneutral plasma physics, atomic physics, and particle physics, and the collaborations include participants from all three fields.

Finally, in his Maxwell Prize Address, Ron Davidson will discuss the Paul Trap Simulator, which uses an electron plasma in a Paul trap (rf trap) to model the dynamics of a beam in an alternating gradient accelerator.

3. All of the nonneutral plasmas that I will be discussing are confined in a Malmberg-Penning trap. The confinement geometry is very simple. A conducting cylinder is divided axially into three sections, with the central section grounded and the two end sections held at positive potential (to confine a positive ion plasma). Also, there is a uniform axial magnetic field. The ion plasma resides in the region of the central grounded cylinder, with axial confinement provided by electrostatic fields and radial confinement provided by the magnetic field. Because the plasma column is unneutralized, Gauss's law requires there to be a radial electric field in the plasma, and this field together with the axial magnetic field produces an **E**x**B** drift rotation of the plasma.

An important property of nonneutral plasmas in these traps is that they can come to a state of global thermal equilibrium and still be confined.[1] This is in contrast to neutral plasmas, which cannot be confined by static electric and magnetic fields and also be in a state of global thermal equilibrium. The thermal equilibrium state is described by the Gibbs distribution, using the Hamiltonian in the rotating frame of the plasma. This distribution determines the gross shape of the plasma and the microscopic order within that shape. For a sufficiently large plasma, one can show that the microscopic order (in the bulk plasma) depends only on the coupling parameter $\Gamma = e^2/akT$, where a is the Wigner-Seitz radius (essentially the interparticle spacing). For $\Gamma \ll 1$, the plasma is weakly correlated, but as the temperature is lowered and Γ is increased, the strength of correlation increases until there is a phase transition to a bcc crystal at about $\Gamma = 174$. For typical densities of trapped nonneutral plasmas, very low temperature (tens of mK) are required to reach such a large value of Γ, and here the atomic physicists come to the rescue with laser cooling of the ions.

4. This slide shows images of a pure ion bcc crystal obtained by John Bollinger and collaborators of the NIST ion storage group in Boulder.[2] The Malmberg-Penning trap is arranged vertically, rather than horizontally. The plasma consists of about 1.8×10^5 singly ionized beryllium atoms at a density of $4 \times 10^8 cm^{-3}$ and a temperature less than 10mK, which

corresponds to a Γ value in excess of 200. The plasma is small (~1.2 mm). The plasma is illuminated by an axial laser-cooling-beam and a transverse laser-cooling-beam, and Fluorescence from the two beams is imaged by a side view camera and a strobed top view camera. The strobe effectively removes the plasma rotation. In the image from the top view camera, one can see light from individual atoms arranged in a bcc crystal. The measured crystal spacing is in good agreement with the predicted spacing.

5. The bcc structure is observed only for a sufficiently large crystal (~ 60 lattice planes). For smaller crystals the competition between the bulk correlation energy and the surface correlation energy leads to a sequence of structural phase transitions. This slide shows a comparison between theoretical predictions by Dan Dubin and measurement by the NIST ion storage group.[3] The abscissa is the projected charge density per unit area (transverse to the crystal planes), the ordinate shows the axial location and number of crystal planes, and the color indicates the crystal structure. At critical values of the projected density, the energy balance shifts in favor of adding a new crystal plane or of changing the crystal structure. The theory (curves) and measurement (points) show good agreement.

6. For the small spheroidal plasmas used by atomic physicists, Dubin obtained an analytic description for all of the plasma modes described by cold fluid theory (a good approximation for these plasmas).[4] This slide shows a comparison between theory and measurement (again by the NIST ion storage group) for the (2,0) and the (9,0) Langmuir modes.[5] In these modes, the ions oscillate back and forth along the magnetic field, and the fluorescence from the axial laser-beam is sensitive to the Doppler shift associated with the oscillation velocity. In a power point presentation, the blue images are movies that show the oscillatory motion. The amplitude and phase of the eigenmodes are obtained from these movies, and are compared to theory in the orange and green images.

   The collaboration between atomic physicists and plasma physicists on these small, laser cooled plasmas and crystals has been a success story. When the collaboration began these plasma systems were rather poorly understood (indeed, they were often referred to as ion clouds, rather than plasmas), and now they are arguably the best-understood and best-controlled plasma systems in existence.

7. To understand why these plasmas model the microscopic order in high-density matter (say in a white dwarf), first note that the electrons in such matter are so Fermi degenerate that they form a nearly rigid, uniform, negative charge density, and

that the ions are pressure ionized classical point charges immersed in that uniform negative charge density. The theoretical model of classical point charges in a uniform neutralizing background charge density is called a one component plasma (OCP) and was studied extensively as a model for high density plasma.[6] One can easily show that the Gibbs distribution for the magnetically confine pure ion plasma differs only by rotation from the Gibbs distribution for a pure ion plasma that is confined by a cylinder of uniform negative charge density.[1] Thus, the magnetically confined plasma is a simple laboratory realization of an OCP, and the observation of the bcc crystal was a confirmation of the structure long predicted for high-density matter.

8. The plasmas also can be used to model thermonuclear fusion in high-density matter. Starting from the observation that Debye shielding reduces the Coulomb repulsion between two ions, thereby allowing the ions to approach one another more closely than they would in the absence of shielding, Salpeter argued that shielding (and more generally correlation effects) enhance the fusion rate.[7] The enhancement factor depends on the coupling parameter Γ, and for the modest values of Γ considered here is approximately $\exp(\Gamma)$. The next two slides report experimental measurement of this Salpeter enhancement factor.

Of course, there are no actual fusion reactions for the cryogenic temperatures of these plasmas, but there is an analogous effect that can stand in for fusion reactions. We say that a plasma is strongly magnetized when the characteristic cyclotron radius is small compared to the classical distance of closest approach, and for such a plasma the cyclotron energy is bound up in an adiabatic invariant that is conserved for most collisions; the cyclotron energy is liberated only in close, energetic collisions.[8] This sounds like fusion, where the nuclear energy is liberated only in close, energetic collisions. Thus, there is an analogy where cyclotron energy plays the role of nuclear energy. Moreover, Dubin showed theoretically that the analogy is quantitative; the rate at which collisions liberate cyclotron energy (the collisional equipartition rate between the parallel and perpendicular velocity components) is enhanced by the same Salpeter factor, $\exp(\Gamma)$.[9] The calculations for the fusion rate and the equipartition rate are completely parallel; in the fusion case, the exponentially small tunneling factor is averaged over a thermal distribution of collisions, and in equipartition case, the exponentially small factor for the breaking of an adiabatic invariant is averaged over a thermal distribution of collisions.

9. This slide shows measurements of the equipartition rate by Anderegg and Driscoll of UCSD. The ordinate is the equipartion rate, the abscissa at the bottom is the plasma temperature, and the abscissa at

the top is the degree of magnetization (cyclotron radius divided by distance of closest approach) for a magnetic field strength of 3 Tesla. The points in blue are measurements for plasma density of $1.2 x 10^6 cm^{-3}$ and the points in red are measurements for plasma density that is 20 times larger. The dashed blue and red curves are the theoretically predicted rates without the Salpeter enhancement factor and the solid blue and red curves are the predicted rates including the enhancement factor. As the temperature is decreased from higher values, the predicted and measured rates first increase as $1/T^{3/2}$ as is expected, but when the regime of strong magnetization is entered ($r_c/b < 1$), the predicted and measured rates begin to drop exponentially. For low density (blue points and curves), the coupling parameter Γ is small even at the lowest temperatures accessed, and negligible enhancement is predicted and observed. However, for the larger density (red points and curves), Γ values larger than unity are accessed and enhancement is predicted and observed. Indeed, the measured enhancement seems to exceed the predicted enhancement. One should note however that these results are very recent and should be considered preliminary.

10. This slide compares the measured enhancement rate to the theoretical prediction exp(Γ). Other more accurate theoretical predictions for the enhancement factor are also included, but are nearly

indistinguishable from exp(Γ) over the limited range of Γ values considered.

11. Fred Driscoll of UCSD developed a method of using pure electron plasmas to model the 2D vortex dynamics and turbulence of ideal fluids (incompressible and inviscid fluids).[10] For these experiments, the trap is operated in an inject-hold-dump sequence. Electrons are produced by a hot spirally wound filament at one end of the apparatus, and are injected into the confinement region and trapped by switching the bias on the near end cylinder to ground and then back to negative. After some time for plasma dynamics, the potential on the far end cylinder is switched to ground, dumping the electrons out along the magnetic field lines to a phosphorous screen, which is imaged by a CCD camera.

The dynamical frequencies are ordered so that the transverse motion of the electrons can be described by bounce-average 2D **E**x**B** drift dynamics. The governing equations for this dynamics (the continuity equation using the drift velocity and Poisson's equation) are identical to the equations for the 2D ideal flow of a neutral fluid, where the electric potential corresponds to the stream function, the drift velocity to the fluid velocity, and the field line averaged electron density (image on the CCD camera) to the vorticity. Thus, the dynamics of the electron plasma models the 2D ideal flow of a neutral fluid.

Moreover, the plasma experiments offer advantages over, say, a water tank experiment. For example, the vorticity is imaged directly (one need not take derivatives of a velocity field), and the plasma flow is characterized by very low vorticity and is not complicated by a boundary layer at the walls.

12. This slide shows how one may create two plasma columns (two vortices). Here, one must realize that the trap wall is more complicated than the three sections mentioned so far; potentials can be applied to various electrically isolated sectors of the wall and the resulting fields can drift the plasma off-axis and can cut the plasma in half axially.

13. This slide shows the merger of two vortices. Each image involves a separate inject-hold-dump sequence, with the hold time increased for the subsequent images. The shot to shot reproducibility is good enough to make a movie of the merger process.

14. Many theoretical studies of 2D ideal flow have predicted that two identical vortices merge when the scaled separation is less than about 1.6. The scaled separation is D/R_p, where D is the separation between vortex centers and R_p is the vortex radius. This slide shows a plot of the measured time to merge versus D/R_p .[11] For D/R_p less than about 1.6, the vortices merge in less than one rotation period, and for D/R_p larger than about 1.6, the time to merge jumps to more than 10^4 rotation periods. This nearly 5 orders

of magnitude increase is a reflection of the very low viscosity for these experiments.

15. This slide shows images of turbulent relaxation, which surprisingly can result in vortex crystal structures.[12] Recall that the electrons are produced at one end by a hot spirally wound filament. If the electrons are trapped suddenly, the trapped plasma is initially an image of the filament, that is, a spirally wound sheet of electrons (or, of vorticity). See the initial states for the top two sequence of images. The spiral is everywhere locally unstable to the Kelvin-Helmholtz instability, so ripples in the spiral grow, saturating nonlinearly in many small vortical structures. During the subsequent, apparently chaotic dynamics the small vortices merge forming fewer but larger vortices. During the merger processes, filamentary tails of vorticity are spun off yielding a low-density background vorticity (green in the images). In some cases (e.g., the second sequence of images), the merger process goes to completion, resulting in a single large vortex. However, a single large vortex is not always the end state. As the vortices move and merge they also stir the background vorticity, and this effectively cools the vortices. Before the merger process goes to completion, the vortices can be annealed into a vortex crystal state, that is, a state of local energy maximum, and this arrests the further evolution. This situation is shown in the first sequence of images. The bottom

set of images is a catalogue of various vortex crystals that resulted from the decaying turbulence.

16. Joel Fajans and his collaborators at UC Berkeley replaced the hot filament electron source with a photocathode and mask, allowing arbitrary initial vortex distributions.[13] The slide shows a good illustration of the flexibility of this technique in the image to the right of the apparatus. My understanding is that this image was created by Joel's graduate students late one night when Joel was not in the lab; as you can see the resembles to Joel is pretty good. In a power point presentation, the three vortex images on this slide are the initial images of movies, which show the vorticity evolution. For example, the image of Joel shears apart through the incompressible flow. The two lower images demonstrate that background vorticity is necessary for the formation of a vortex crystal. For the collection of vortices in the image on the left, there is no background vorticity, and the subsequent evolution does not lead to a crystal. Whereas, in the image on the right, the same set of vortices is immersed in a distribution of background vorticity, and the subsequent evolution does lead to a vortex crystal. The vortices are effectively cooled and annealed into a crystal by stirring the background vorticity.

17. Another analogy is between 2D ideal flow and plasma physics arises from the fact that both 2D ideal flow and Vlasov flow (say in a 2D phase space) are

incompressible flow. For the 2D ideal flow, the continuity equation in the (r,θ) configuration space can be written as a Vlasov equation for the flow in the guiding center phase space ($p_\theta = eBr^2/2c$, θ). In other words, the configuration space can be re-interpreted as a phase space, and phenomena that we associate with Vlasov dynamics (e.g., Landau damping and plasma wave echoes) also appear in the 2D ideal flow.

18. This slide shows experimental results from Fred Driscoll and collaborators at UCSD. The three images at the top left show a vortex on which an m=2 Kelvin wave (or Diocotron wave) has been excited. The wave propagates azimuthally around the vortex and the vortex also rotates azimuthally, with a radially dependent drift rotation frequency, $\omega_{drift}(r)$. At a certain critical radius r_c , the wave phase velocity and the fluid rotation velocity match, that is, there is a spatial Landau resonance, $\omega = l\,\omega_{drift}(r_c)$. The plot below shows the amplitude of the wave (the quadrupole moment of the vortex, for this m =2 wave) versus time. Initially, the wave undergoes linear Landau damping, and then the damping saturates nonlinearly and the wave amplitude exhibits trapping oscillation. Of course, nearly identical plots were obtained years ago for the Landau damping and nonlinear trapping oscillations of 1D plasma waves.

19. This slide shows experimental images of fluid echoes obtained by the UCSD group. The echoes are the

analogue of plasma wave echoes in 1D Vlasov dynamics. The four images (a through f) show the vorticity perturbation (obtained through a subtraction procedure) at a sequence of times. The corresponding signal measured on a wall probe is shown below these four images. Image a shows the vorticity perturbation immediately after an m=2 Kelvin wave has been excited. Images b and c show the phase mixing of this perturbation due to shear in the rotational flow, and the trace below shows the corresponding Landau damping of the wall signal. Image d shows the perturbation immediately after an m=4 Kelvin mode has been excited. The wall signal for this mode also Landau damps away as the vorticity perturbation phase mixes. However, in second order, the m=4 wave modulates the phase mixed pattern from the m=1 wave, and this second order perturbation begins to un-phase mix, producing the echo by image f.

For the plasma wave echo, the phase mixing cannot be observed directly since it occurs in phase space—a mathematical construct. However, for the fluid echo, the phase space also happens to be the configuration space so the phase mixing can be observed directly.

References:

1. Daniel H. E. Dubin and T. M. O'Neil, Rev. Mod. Phys. **71**, 87 (1999)

2. J.J. Bollinger, T. B. Mitchell, X. P. Huang, W. M. Itano, J. N. Tan, B. M. Jelenkovic, and D. J. Wineland, Phys. Plasmas **7**, 7 (2000)
3. T. B. Mitchell, J. J. Bollinger, D. H. E. Dubin, X. P. Huang, W. M. Itano, and R. H. Baughman, Science **282,** 1290 (1998)
4. D. H. E. Dubin, Phys. Rev. Lett. **66**, 2076 (1991)
5. T. B. Mitchell, J. J. Bollinger, X. P. Huang, and W. M. Itano, Optics Express **2**, 314 (1998)
6. S. Ichimaru, Rev. Mod. Phys. **54**, 1017 (1982)
7. E. E. Salpeter and H. M. Van Horn, Astrophys. J. **155**. 183 (1969)
8. M. E. Glinsky, T. M. O'Neil, M. N. Rosenbluth, K. Tsuruta, and S. Ichimaru, Phys. Fluids B **4**, 1556 (1992)
9. D. H. E. Dubin, Phys. Rev. Lett. **94**, 025002 (2005)
10. C. F. Driscoll and K. S. Fine, Phys. Fluids B **2**, 1359 (1990)
11. K. S. Fine, C. F. Driscoll, J. H. Malmberg, and T. B. Mitchell, Phys. Rev. Lett. **67**, 588 (1991)
12. K. S. Fine, A. C. Cass, W. G. Flynn, and C. F. Driscoll, Phys. Rev. Lett. **75**, 3277 (1995)
13. D. R. Durkin and J. Fajans, Rev. Sci. Instrum. **70**: 4539 (1999)