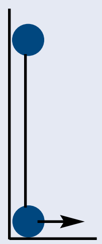
Physics Challenges for Teachers and Students

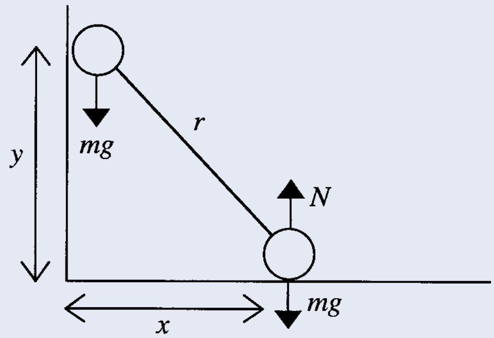
Solutions to the December Challenges

How Low Can You Go?

Challenge: A dumbbell consists of a light rod of length *r* and two small masses *m* attached to it. The dumbbell stands vertically in the corner formed by two frictionless planes. After the bottom end is slightly moved to the right, the dumbbell begins to slide. Find the speed *u* of the bottom end at the moment the top end loses contact with the vertical plane.



Solution: Since the normal force on the top mass is zero and the horizontal acceleration of the top mass is zero at the instant it loses contact with the wall, the tension in the rod at that moment must be zero. Thus, free-body diagrams for the two masses are as sketched below.



Therefore, the top mass has downward velocity $v = -dy/dt$ and acceleration $g = -d^2y/dt^2$, while the bottom mass has rightward velocity $u = dx/dt$ and zero acceleration. But $y = \sqrt{r^2 - x^2}$ since the length of the rod is fixed, and thus

$$v = -\frac{dy}{dt} = \frac{x}{\sqrt{r^2 - x^2}}\frac{dx}{dt} = \frac{xu}{y}$$

and

$$g = -\frac{d^2y}{dt^2} = \frac{u}{y}\frac{dx}{dt} - \frac{xu}{y^2}\frac{dy}{dt} = \frac{u^2}{y} + \frac{xuv}{y^2}$$

$$= \frac{y^2u^2}{y^3} + \frac{x^2u^2}{y^3} = \frac{r^2u^2}{y^3}.$$

Finally, from conservation of mechanical energy,

$$mg(r - y) = \frac{1}{2}m(u^2 + v^2) \Rightarrow 2g(r - y) =$$

$$u^2\left(1 + \frac{x^2}{y^2}\right) = \frac{u^2r^2}{y^2} = gy$$

so that

$$y = \frac{2}{3}r \Rightarrow u = \sqrt{\frac{8gr}{27}}.$$

(Contributed by Carl E. Mungan, U.S. Naval Academy, Annapolis, MD)

The Sled that Almost Fled

Challenge: A sled is given a quick push up the snowy slope. The sled slides up and then comes back down; the whole trip takes time *t*. If the coefficient of sliding friction between the sled and the snow is μ, find the time t_u it took the sled to reach the top point of its trajectory. The slope makes the angle θ with the horizontal.

Solution: Newton's second law applied to the sled gives accelerations upwards (positive direction up) and downwards (positive direction down) as follows: $a_u = -g(\sin\theta + \mu\cos\theta)$ and $a_d = g(\sin\theta - \mu\cos\theta)$. If u and v are, respectively, the initial velocity when moving upwards and the final velocity when moving downwards, then according to the (constant acceleration) kinematical formula involving squares of velocities, $a_u/a_d = -(u/v)^2$. If t_u and t_d are, respectively, the time it takes to move up and the time it takes to move down, then according to the definition of acceleration and using the previous equation, $t_u/t_d = (-a_d/a_u)^{1/2}$.

Combining the latter equation with the first two, we get (noting that tan $\theta > \mu$ since the sled comes back):
$t_u = t/[1 + \sqrt{(\tan\theta + \mu)/(\tan\theta - \mu)}]$.

(Contributed by Inge H. A. Pettersen, Kongshavn, Norway)

Ups and Downs at Work

Challenge: The elevator operator has an eight-hour shift to work. She wants to be sure that she works *exactly* eight hours and installs a pendulum clock in the elevator. Is the operator going to achieve her goal? Assume that the upward and downward accelerations of the elevator have the same magnitude and that the times of accelerating upward and downward are the same, according to a *resting* clock.

Solution: The apparent time interval recorded by a pendulum clock is proportional to the number of swings of its pendulum, which is proportional to the pendulum's frequency. That frequency, in turn, is proportional to the square root of the effective gravitational field strength in the elevator. Let αg represent the elevator's acceleration. The effective gravitational field strength in the elevator's frame of reference is then $g_u = g(1 + \alpha)$ when accelerating upward and $g_d = g(1 - \alpha)$ when accelerating downward.
Let T represent the true time spent accelerating upward; an equal amount of time would be spent accelerating downward, so the total accelerating time is $2T$. Also, let t_u be the time spent accelerating upward *as recorded by the pendulum clock*; t_d ("down") can be defined similarly.
Let us find the ratio R of the total time recorded by the pendulum clock to the total true time that the elevator spends accelerating:

$$R = (t_u + t_d)/(2T) = [(g_u)^{1/2} + (g_d)^{1/2}]/[2g^{1/2}] =$$

$$[(1 + \alpha)^{1/2} + (1 - \alpha)^{1/2}]/2.$$

We can simplify the equation by squaring both sides:

$$R^2 = [(1 + \alpha)^{1/2} + (1 - \alpha)^{1/2}]^2/4$$

$$= [2 + 2(1 - \alpha^2)^{1/2}]/4 = ½ + [½\,(1 - \alpha^2)^{1/2}] .$$

It is now clear that the ratio is less than 1, showing that the time recorded by the pendulum clock will be less than the true time, and the elevator operator ends up working overtime.
(Contributed by Art Hovey, Milford, CT)

Other Contributors

Several other readers also sent us some solutions to the December Challenges. We would like to recognize the following contributor:

John F. Goehl Jr. (Barry University, Miami Shores, FL)

We appreciate your submissions and hope to receive more solutions in the future.

Note to Contributors:

As the number of submissions grows, we request that certain guidelines be observed, in order to facilitate the process more efficiently:

- Please email the solutions as Word files.
- Please name the file as "April03HSimpson" if — for instance — your name is Homer Simpson, and you are sending the solutions to April 2003 Challenges.
- Please state your name, hometown, and professional affiliation in the file, not only in the email message.

Many thanks!

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