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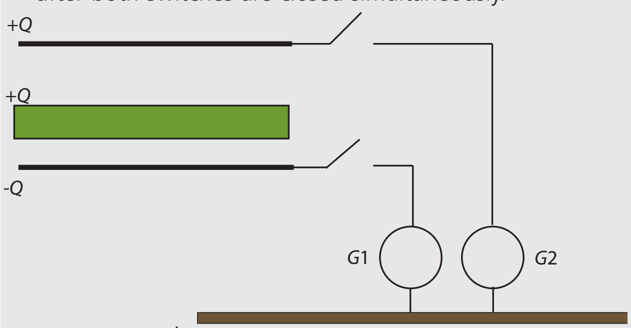
Physics Challenge for Teachers and Students

Solution to February 2009 Challenge

From the Gound Up and Down

A parallel-plate capacitor is charged as shown (Q is given). A metal slab with the total charge $+Q$ is placed inside the capacitor as shown. The thickness of the slab is d. The distance between the top plate and the top of the slab is $2d$, and the distance between the bottom plate and the bottom of the slab is d. Each plate is grounded through a galvanometer as shown.

Find the charge that passes through each galvanometer after both switches are closed simultaneously.



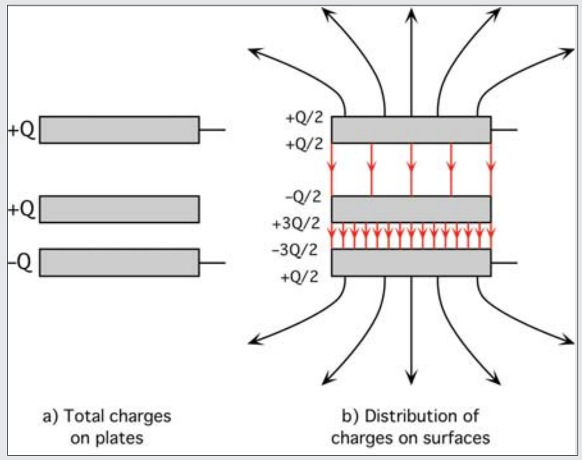


Fig. 1. Initial situation.

Solution:

We must assume that all facing surfaces have equal areas and are large compared to their separation so that we can ignore fringing fields from the facing surfaces. With this assumption we know that the charges on facing surfaces will be equal in magnitude and opposite in sign.

Because the entire, three-plate capacitor assembly is initially isolated and carries a total charge of +Q as shown in Fig. 1(a), there will be field lines leaving the outer surfaces. If we assume that the ground plane is relatively far away, then the external field will be roughly symmetric and we will have +Q/2 on the uppermost and lowermost surfaces. The charges on the other surfaces are then completely determined as shown in Fig. 1(b) along with a representation of the fields that they would produce.

Note that the top plate has a higher potential than the middle plate, which, in turn, has a higher potential than the bottom plate. We can also see that the field between the upper-facing surfaces is 1/3 that between the lower-facing surfaces. Finally, since the upper-facing surfaces are twice as far apart as the lower-facing surfaces, their potential difference will be 2/3 that of the lower-facing surfaces.

With all of that understood, the final situation can be quickly determined by inspection. We need only keep in mind that:

1. Grounding the top and bottom plates brings them to the same potential and eliminates the excess charge on the outer surfaces (since there can be no external field between the assembly and the ground plane).
2. The middle plate retains its charge, i.e., +Q, because it remains isolated.
3. In order to make the potential difference be-

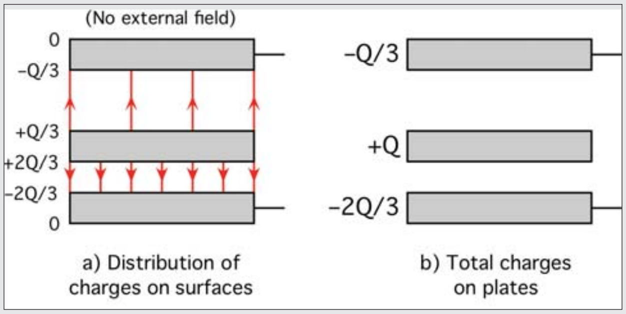


Fig. 2. Final situation.

tween the middle plate and the top plate the same as that between the middle plate and the bottom plate, the field between the upper-facing surfaces must be 1/2 that between the lower-facing surfaces.

4. Thus, the charge on the top surface of the middle plate must be 1/2 that on the bottom surface, i.e., +Q/3 on the top surface and +2Q/3 on the bottom surface.

The solution is shown in Figure 2.

Finally, comparing the initial and final charges on the plates, we see that:

1. a total charge of -Q/3 flows (through G1) from the bottom plate to ground and
2. a total charge of +4Q/3 flows (through G2) from the top plate to ground.

Note that the net charge flowing to ground, +Q, is equal to the initial excess charge on the three-plate capacitor assembly as it must be.

(Submitted by John Mallinckrodt, Cal Poly Pomona, Pomona, CA)

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