Physics Challenge for Teachers and Students

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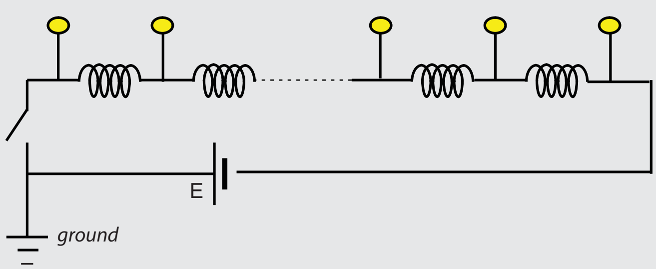
Solution to March 2009 *Challenge*

The Long March of 2009

Challenge: A circuit contains an emf source E with negligible internal resistance and 2008 identical resistors. 2009 small identical conducting spheres are connected to the circuit by long thin wires as shown. When the switch is closed, the *total* charge of the spheres changes by *Q*. What is the radius *r* of each sphere?



Solution: After closing the switch, we experience a transient, time-dependent current in the circuit, but after a long time a constant dc current will flow in the circuit, and the charge distribution on the spheres will not change; therefore the spheres will not affect the current in the main loop. Theoretically, each sphere forms a capacitor with the ground being the other plate (self-capacitance), and another capacitor with any of the remaining spheres (mutual-capacitance). In the following we neglect the mutual-capacitance [theoretically $n(n-1)/2$ capacitances, if we have n spheres]. The self-capacitance of a conductor sphere with radius r is

$$C = 4\pi\varepsilon_0 r. \quad (1)$$

Moreover, by the definition of the capacitance we can relate the radius of the sphere and the potential-difference across the capacitance:

$$C = \frac{Q_n}{U_n}, \quad (2)$$

where Q_n denotes the charge of the nth sphere and U_n is the potential-difference between the ground and the nth sphere. Let us number the spheres starting from the left-most sphere, which we call the zeroth sphere and the last one is N.

$$CU_n = Q_n \quad (3)$$

$$\sum_{n=0}^{N} CU_n = \sum_{n=0}^{N} Q_n. \quad (4)$$

Since the capacitance of each sphere is the same, we can pull C out of the summation, obtaining

$$C\sum_{n=0}^{N} U_n = Q. \quad (5)$$

After a long time the dc current in the circuit will be

$$I = \frac{E}{NR}. \quad (6)$$

And therefore the potential of the nth small sphere is

$$U_n = nIR = n\frac{E}{NR}R = \frac{E}{N}n. \quad (7)$$

Substituting (7) into (5) we can eventually derive the radius of the spheres

$$r = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sum_n U_n} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\frac{E}{N}\sum_n n}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{NQ}{E\frac{N(N+1)}{2}} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{E}\frac{1}{N+1}$$

$$r = \frac{1}{2\pi\varepsilon_0}\frac{Q}{E}\frac{1}{N+1}, \quad (8)$$

where $N = 2008$.

(Submitted by Daniel Schumayer, University of Otago, Dunedin, New Zealand)

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Many thanks to all contributors and we hope to hear from you in the future!

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