# Physics Challenge for Teachers and Students

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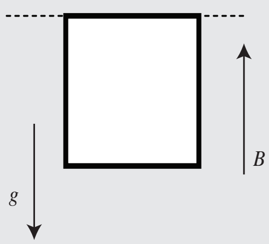
## Solution May 2010 Challenge

### Be There *and* Be Square

A square loop with side *b* is made of a wire of mass *m* and negligible electric resistance. The loop is pivoted along its top horizontal side and placed in the weak vertical uniform magnetic field *B* as shown. The loop is then pulled to a horizontal position and released. Eventually, the loop comes to rest due to air resistance. Find the angle $\theta$ that the plane of the loop makes with the vertical at the final position. The inductance of the loop is *L*.

*(Adapted from* Physics Olympiads *by A. Slobodyanjuk, L. Markovich, A. Lavrinenko. Aversev, Minsk, 2003.)*




The solutions to the May *Challenge*, **Be There *and* Be Square**, came from all over the world. They were not quite as numerous as we had expected. One possible reason is that the problem was poorly worded and thus unduly confusing (the column editor accepts full responsibility and humbly apologizes for the mishap). A small number of attentive readers noted that, as stated, the problem was of no particular interest and they proposed an alternative version for the initial conditions (which was, incidentally, the intended one). Interestingly, the vast majority of the successful solvers simply interpreted the problem the way it was intended (but not actually stated) without explicitly explaining why they chose to alter the initial conditions.

Also, a number of contributors stopped short of a full solution by providing the equation but not solving it. Using the condition of "weakness" of the magnetic field, the approximate solution could be found rather easily; if no attempt to arrive at the final answer was made, the solutions were considered incomplete.

Here is one of the solutions that explains why the problem should be rephrased and then proceeds to solve the altered version:

***Solution***: The key to the problem is to note that the total emf $\varepsilon$ induced in the loop is the negative time derivative of the total magnetic flux $\Phi$ linked by the loop,

$$\varepsilon = -\frac{d\Phi}{dt} = IR \tag{1}$$

using Ohm's law in the last step, where *I* is the current induced in the loop. But since the resistance *R* of the loop is zero, we conclude that $\Phi$ must be a constant. Now in general that total flux can be written as a sum of the flux $\Phi_{ext}$ due to external magnetic field *B* and the flux $\Phi_{ind}$ in the loop due to the induced current *I*. Specifically,

$$\Phi = Bb^2 \sin\theta' + LI \tag{2}$$

at an arbitrary instant in time when the loop hangs at angle $\theta'$ with respect to the vertical. (We are told that $\theta'$ oscillates for a while and finally stops at value $\theta$.) I have chosen the sign convention that upward flux is positive and hence current is positive if it is *counterclockwise* as seen looking down on the loop from above (in accord with the right-hand rule). Since $\Phi$ is a constant, we can determine its value from the initial conditions in the problem. We are told the loop was initially hanging vertically at rest (before being pulled up to a horizontal position) so that $\theta'_i = 0$ and presumably the initial current in the loop was zero, $I_i = 0$. In that case, $\Phi$ is initially zero, as we see from Eq. (2), and consequently must always be zero. But that means that when we pull the loop up into a horizontal position ($\theta' = 90°$), a *clockwise current* equal to $-Bb^2/L$ is induced in the loop, according to Eq. (2). Now consider the following sketch of the loop just after it is released from this position and begins to fall.

Only the current along the edge farthest from the axis (indicated by the dashed line above) contributes to the