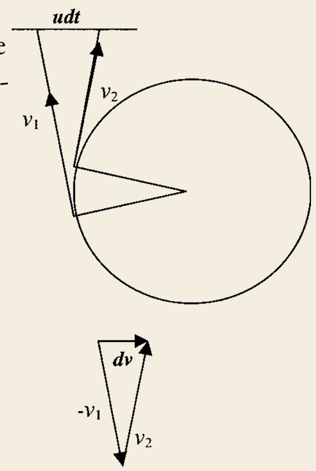
Physics Challenges for Teachers and Students

Solutions to October 2002 problems.

◗ Cats and Dogs



The cat runs along the straight path with constant velocity. The dog runs toward the cat with speed v. When the two velocities are perpendicular, the distance between the dog and the cat is x. What is the dog's acceleration at that time? The diagram shows the dog's velocity just before and just after the moment in question, with a very small angle between them. Extending those arrows to the path of the cat forms an isosceles triangle with height x and width udt. That triangle is similar to the bottom one, formed when we subtract the two velocities to obtain the change in velocity dv. From the similarity of these triangles, we have

$$udt/x = \mathrm{d}v/v, \quad \text{so} \quad dv/dt = uv/x$$

(Contributed by Art Hovey, Milford, CT)

◗ The Cable News

Let x be the varying length of the cable on the horizontal table. The cable's acceleration is provided by the weight of the vertical part only. Considering the whole cable as a system of mass M, we can write: $M[1–(x/L)]g = Ma$. The acceleration is then $a = (1 – x/L)g$. For the horizontal part, the acceleration is provided by the tension at the bending point. Therefore, the force of tension on the horizontal part is $\mu xa = \mu x(1– x/L)g$, where μ is the linear mass density. This force has to be less than the maximum tension force μgl, i.e., $0 < x^2 – xL + Ll$. Thus, the discriminant of the quadratic function on the right-hand side has to be negative, leading to $L < 4l$.

(Contributed by Inge H. A. Pettersen, Kongshavn, Norway)

◗ Slippery Slope

The acceleration and time up the incline are given by the following equations:

$$a_u = -g(\sin\theta + \mu\cos\theta)$$
$$t_u = -v_o/a_u = v_o/g(\sin\theta + \mu\cos\theta), \qquad (1)$$

where v_o is the initial velocity of the object moving up the incline. Similarly, for the object moving down the incline,

$$a_d = g(\sin\theta - \mu\cos\theta)$$
$$t_d = v/a_d = v/g(\sin\theta - \mu\cos\theta), \qquad (2)$$

where v is the final velocity of the object at the bottom of the incline. Dividing Eq. (1) by Eq. (2) yields

$$t_u/t_d = \mu = \frac{v_o(\sin\theta - \mu\cos\theta)}{v(\sin\theta + \mu\cos\theta)}. \qquad (3)$$

From kinematics equations:

$$v_o = (2a_u x)^{1/2} \text{ and } v = (2a_d x)^{1/2}.$$

After these substitutions, Eq. (3) becomes

$$\mu = \frac{(\sin\theta + \mu\cos\theta)^{1/2}(\sin\theta - \mu\cos\theta)}{(\sin\theta - \mu\cos\theta)^{1/2}(\sin\theta + \mu\cos\theta)},$$

which can be simplified as:

$$\mu = \frac{(1 - \mu \cot \theta)^{1/2}}{(1 + \mu \cot \theta)^{1/2}}. \quad (4)$$

Solving Eq. (4) for θ yields:

$$\cot \theta = \frac{(1 - \mu^2)}{\mu(1 + \mu^2)}$$

or, more conventionally,

$$\tan \theta = \frac{\mu(1 + \mu^2)}{(1 - \mu^2)}.$$

Note that since the right side of the equation is always greater than μ, $\tan \theta > \mu$, and the condition of slipping is satisfied.

(Contributed by Robbie F. Kouri, Our Lady of the Lake University, San Antonio, TX)

• **Several other readers** also sent us correct solutions to the October Challenges. We would like to recognize the following contributors:

Dylan Consla, student (Maine School of Science and Mathematics, Limestone, ME)

Rusty Davis (Taft School, Watertown, CT)

Carl E. Mungan (United States Naval Academy, Annapolis, MD)

Inge H. A. Pettersen (Kongshavn, Norway)

We appreciate your submissions and hope to receive more solutions in the future.

Note to contributors: As the number of submissions grows, we request that certain guidelines be observed, in order to facilitate the process more efficiently:

– Please e-mail the solutions as Word files;
– please name the file as “Jan03HSimpson” if, for instance, your name is Homer Simpson and you are sending the solutions to January 2003 Challenges;
– please state your name, hometown, and professional affiliation in the *file*, not only in the e-mail message.

Many thanks!

Please send correspondence to:

Boris Korsunsky,
444 Wellesley St., Weston,
MA 02493-2631;
korsunbo@gse.harvard.edu