Physics Challenge for Teachers and Students

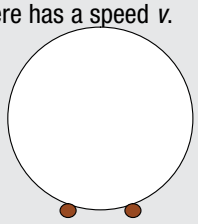
Boris Korsunsky, **Column Editor**
Weston High School, Weston, MA 02493
korsunbo@post.harvard.edu

Solutions to September 2011 Challenge

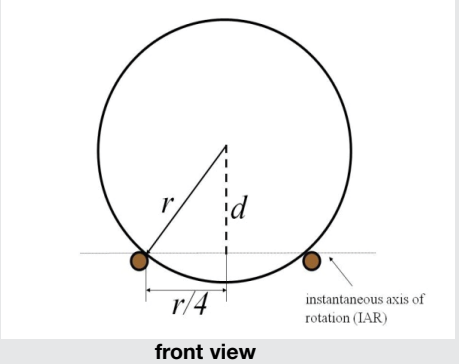
Slipless in Seattle

A sphere of radius r is rolling horizontally without slipping on two parallel rails placed a distance $r/2$ apart (in the diagram the sphere is rolling perpendicular to the plane of the page).

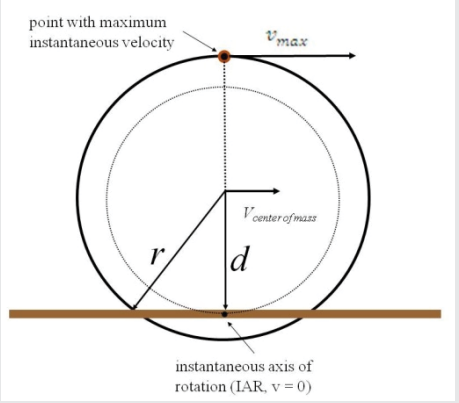
Which point(s) of the sphere have the maximum instantaneous velocity? Find that maximum velocity if the center of mass of the sphere has a speed v.



Solution: The line connecting the rails is the instantaneous axis of rotation (front view in the figure). We are going to calculate the value of d (see the figure).



front view



side view

$$\left(\tfrac{r}{4}\right)^2 + d^2 = r^2 \qquad (1)$$

We can obtain the value of d from Eq. (1):

$$d = \frac{\sqrt{15}}{4} r \qquad (2)$$

The point with the maximum instantaneous velocity would be the point of the sphere with the largest distance from the instantaneous axis of rotation (side view in the figure). This point is the top of the sphere.

We can write a relationship between the velocity of the center of mass and the angular velocity (ω) with respect to the IAR:

$$v = d\omega \qquad (3)$$

Using Eqs. (2) and (3), we obtain the value of ω:

$$\omega = \frac{4v}{\sqrt{15}r} \qquad (4)$$

The point with the maximum instantaneous velocity has the same ω with respect to the IAR as the center of mass. Therefore, we can calculate the velocity of the point with maximum velocity as:

$$v_{\max} = \omega(r + d) \qquad (5)$$

Finally, we obtain

$$v_{\max} = \left(\frac{15 + 4\sqrt{15}}{15}\right) v$$

(Contributed by Israel Pérez Luna, student, Escuela Politécnica Superior, Seville, Spain)

We would also like to recognize the following contributors:

Sharmila Balamurugan (undergraduate student, Women's Christian College, Chennai, India)

Dave Bittel (Bristol Eastern High School, Bristol, CT)

Phil Cahill (Lockheed Martin Corporation, North Yorkshire, United Kingdom)

Bartley L. Cardon (MIT Lincoln Laboratory (retired), South Portland, ME)

Juan Carlos Criado Alamo (student, University of Granada, Spain)

Don Easton (Lacombe, Alberta, Canada)

Fernando Ferreira (Universidade da Beira Interior, Covilhã, Portugal)

Josh Gates (Tatnall School, Greenville, DE)

Norge Cruz Hernández (University of Seville, Spain)

Harold Houghton (Maggie Walker HS, Richmond VA)

Art Hovey (retired, Milford, CT)

José Ignacio Íñiguez de la Torre (Universidad de Salamanca, Salamanca, Spain)

David Jones (Florida International University, Miami, FL)

Mark Lenfestey (Homestead High School, Fort Wayne, IN)

Damien Martin (Whittier College, Los Angeles CA)

Daniel Mixson (Naval Academy Preparatory School, Newport, RI)

Carl E. Mungan (U. S. Naval Academy, Annapolis, MD)

Clark M. Neily, Jr. (Hermon's of Alaska Christian School, Allston, MA)

Pascal Renault (John Tyler Community College, Midlothian, VA)

Jason L. Smith (Richland Community College, Decatur, IL)

Clint Sprott (University of Wisconsin – Madison, WI)

William J. Wilde (Northampton Area Senior High School, Northampton, PA)

Many thanks to all contributors and we hope to hear from you in the future!