Fermi Questions

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Question 1: Running down court

How far does a basketball player travel during a regulation basketball game?

Answer: We can estimate the distance travelled by a basketball player either from his average speed or from the number of times that he travels from one end of the court to the other. Let's try both methods and compare the results.

While a basketball player is in motion, his speed is between 1 m/s (a moderate walking pace) and 10 m/s (a world-class sprint). We'll take the geometric mean of 3 m/s (a slow run). However, basketball players are only in motion about half the time (more than one-quarter and less than 100%).

A basketball game lasts 40 minutes so that the average distance travelled is

$$d = (40 \text{ min})(60 \text{ s/min})(3 \text{ m/s})(0.5) = 4 \text{ km}.$$

That is a reasonably long distance.

Let's estimate the number of times they travel up and down the court. Professional basketball games frequently have scores of about 100 on each side. At 2 points per basket, this means that each player has to travel from one end of the court to the other and back 50 times. (Note that this ignores the effects of 3-point shots and foul shots which decrease the number of round trips and of missed baskets which increase the number of round trips.) A basketball court is about 25 m from one end to the other. This means that the distance travelled will be

$$d = (50 \text{ round trips})(50 \text{ m/r–t}) = 3 \text{ km}.$$

Thus the estimates agree (at least within the expected precision of the estimates).

Question 2: Cool It!

How much cooling water do American electrical power generators use each year?

Answer: Almost all of our electrical power is generated by either oxidizing fossil fuel or fissioning uranium to release energy and boil water. The steam pressure is then used to move magnets near coils of wire (or to move coils of wire near magnets) and generate a potential difference. According to the second law of thermodynamics, less than 100% of the thermal energy of the steam can be turned into mechanical or electrical energy. The unused thermal energy must be disposed of, typically by water cooling.

Therefore, in order to answer this question we need to estimate the electrical energy generated each year, the thermal energy needed to generate the electrical energy, and the energy that water can absorb.

We can estimate the electrical energy generated in a few ways. A typical household power bill is about \$100 per month (more than \$10 and less than \$1000) and the cost per kiloWatt-hour is about \$0.10 (more than a penny and less than a dollar). This means that a household uses 10^3 kW-hr per month or about 10^4 kW-hr per year. The 3×10^8 Americans consist of about 10^8 households, so together we use 10^{12} kW-hr per year. Let's double this to include commercial and industrial uses. At 4×10^6 J/kW-hr, this gives a total yearly electrical energy consumption for the United States of

$$E_e = (2\times10^{12}\,\text{kW-hr})(4\times10^6\,\text{J/kW-hr}) = 10^{19}\,\text{J}\,.$$

Alternatively, you might remember that a typical nuclear power plant generates 10^9 W, and that the 100 nuclear power plants supply about 20% of our electrical energy. This gives a yearly energy consumption of

$$E_e = 5(10^2)(10^9\,\text{W})(\pi\times10^7\,\text{s}) = 2\times10^{19}\,\text{J}.$$

These two estimates are reasonably close, so let's use $E_e = 10^{19}$ J.

The typical thermal efficiency is more than 10% and less than 100%, so we will take the geometric mean and estimate 30%. This means that we need to consume $E_t = 3E_e = 3\times10^{19}$ J and that we will need to dispose of $E_{\text{waste}} = E_t - E_e = 2\times10^{19}$ J. That is a LOT of energy to get rid of. Converting Joules to MegaTons of TNT, we get

$$E_{\text{waste}} = 2\times10^{19}\,\text{J}\,\frac{1\text{ MT}}{4\times10^{15}\,\text{J}} = 5\times10^{3}\text{ MT}. \qquad \text{Ouch!}$$

Now let's estimate the cooling capacity of water. We can either heat the water or boil the water. In the first case we need to estimate the heat capacity of water and in the second we need to estimate the latent heat of vaporization. Fortunately, the heat capacity of water is a basic constant, $L = 1$ cal/g-K. Unfortunately, we need to remember the conversion from calories to Joules (1 cal = 4 J). If we heat each gram of water by 50 K (from 25° to 75°C), then we will need

$$m = \frac{2\times10^{19}\,\text{J}}{(50\ \text{K})(4\ \text{J/cal})(1\ \text{cal/g-K})} = 10^{17}\,\text{g} = 10^{11}\,\text{tons}$$

of water. This is a volume of water that is

$$V = 10^{11}\text{m}^3 = 100\ \text{km}^3.$$

That is a huge amount of water!

Let's try boiling the water instead. When you boil water in a tea kettle, it takes much more time to boil away all the water than it does to heat the water from room temperature to the boiling point. If it takes 5 minutes to boil the water in a tea kettle, it will take more than 5 minutes and less than an hour to boil it dry. Thus, we will estimate that the latent heat of vaporization is about three times larger than the energy needed to bring the water to a boil. The energy needed to raise 1 g of water from 20° to 100°C is 80 cal = 300 J. Therefore the latent heat of vaporization is 10^3 J/g.

This means that if we use those large cooling towers to evaporate the water, we will only need to evaporate

$$m = \frac{2\times10^{19}\,\text{J}}{(10^3\ \text{J/g})} = 2\times10^{16}\,\text{g} = 2\times10^{10}\,\text{tons}$$

of water, or about 20 km^3. That is still a LOT of water.

A single 1-GW power plant will need

$$m = \frac{(2\times10^{9}\,\text{W})(\pi\times10^{7}\,\text{s})}{(10^3\ \text{J/g})} = 6\times10^{13}\,\text{g} = 6\times10^{7}\,\text{tons}$$

of water. This is a volume that corresponds to a 10 km^2 lake with a depth of 6 m.

This is why large power plants are placed on rivers or large bodies of water.