FIG. 1. Mixing state of an initially segregated drop of ink in a two dimensional stirring field.

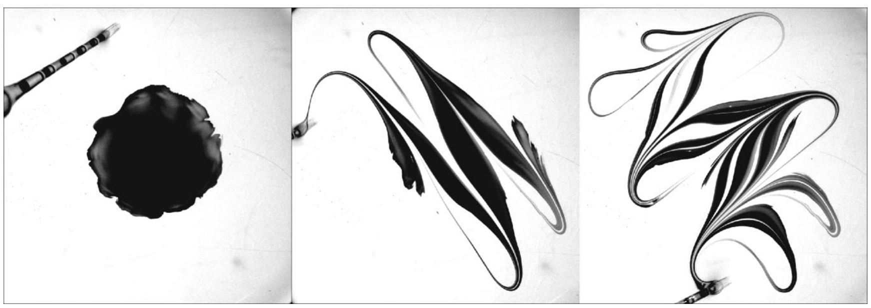

FIG. 2. The stirring protocol of a drop of ink deposed at the surface of pure Glycerol using a small rod. The sequence displays the initial state, half, and a completed stirring cycle.

Mixture's Route to Uniformity by Coalescence

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The paradigm of the spoon stirring a drop of milk in a cup of coffee is often referred to when discussing mixing. We present the viscous version of it: the straw in a milkshake. Stirring a blob of dye with a rod in a thin layer of a viscous fluid (Fig. 1) is instructive to understand how a mixture evolves towards uniformity.

The stirring protocol consists in slicing the medium in the plane of the fluid layer with a small rod. A number of parallel cuts is made in one direction, and then the same number at a right angle, this operation defining one cycle (Fig. 2).

At low Reynolds number (typically 10^{-1}) the medium is deformed by the rod on a scale given by its own size. The maximal rate of stretch is obtained for fluid particles close to the rod trajectory, while the protocol leaves nearly unstretched fluid parcels far from its trajectory. Those keep a concentration close to the initial concentration before being, possibly, stretched at the next cycle. Concomitantly, fluid particles are brought close to each other in the wake of the rod and coalesce. Coalescence results in the addition of the concentration levels of nearby particles and the protocol is such that this addition process is made at random. The mixture concentration distribution $P(C)$ thus evolves by *self convolution* and is actually found to be well described by a family of Gamma functions $P(C)=(n^n C^{n-1} e^{-n(C/\langle C\rangle)})/(\langle C\rangle^n \Gamma(n))$.

Since material lines grow in proportion to the number of cycles in this two-dimensional flow, the maximal concentration of one stretched, diffusing particle decays like (number of stirring cycles)$^{-[1+(1/2)]}$. The number of convolutions keeping the average concentration $\langle C\rangle$ constant cycle after cycle is thus $n \propto$(number of stirring cycles)$^{3/2}$, leading eventually to the mixture's uniformity $P(C,n\rightarrow\infty) \rightarrow \delta(C-\langle C\rangle)$.